MASS TRANSFER INTO THE LIQUID IN TURBULENT FLOW AT HIGH SCHMIDT NUMBERS. THEORETICAL SOLUTION

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Theoretical relation is derived describing the mass transfer into the liquid in turbulent flow with steady velocity profile, at high Schmidt numbers $Sc \geqq 10^3$ in the concentration entrance section. Graphical dependences describing the effect of parameters of the model on mass transfer coefficients are presented and discussed. It is demonstrated that for the limiting cases the derived theoretical solution transforms into familiar relations (asymptotes).

In theoretical studies concerning the mass transfer mechanism across the interface, the hydrodynamics at the interface and in its close vicinity must be known. But the exact mathematical description of hydrodynamics in the given region is in majority of actual situations not applicable. Thus at description of the phenomena it is necessary to start from simplified models which enable to obtain the mathematical expression in a solvable form. The application of various simplifying assumptions has led to formation of a number of models on mass transfer mechanism such as the film theory, penetration theory, theories based on the assumption of eddy diffusivity and others.

From the analysis of the available literature has resulted that none of the published models is capable to describe with a sufficient accuracy the mass transfer mechanism in the whole real range of Schmidt numbers. In this respect the widest range is covered by the model published by Kolář[1] which has been already verified experimentally for the range of Schmidt numbers $3 \cdot 10^{-3} \leqq Sc \leqq 3 \cdot 10^3$. Thus it has appeared appropriate to verify its validity also in the remaining region of high Schmidt numbers ($Sc > 3 \cdot 10^3$).

In this considered region the low diffusivity results in a considerable prolongation of the entrance section necessary for formation of a steady concentration profile *i.e.* of the situation which can be described by the theoretical model[1]. For values $Sc \geqq 10$, follows from the proposed model for the dimensionless mass transfer coefficient in steady state k_∞^+ the relation

$$k_\infty^+ = \left[\left(\lambda^+ \sqrt{\pi}/2 + \delta^+ \, Sc^{1/2}\right) Sc^{1/2}\right]^{-1} . \qquad (1)$$

For very large values of the Schmidt number it is possible to neglect the left hand side term in the parentheses of the relation (*1*) so that the relation is obtained

$$k_{\infty}^{+} = 1/(\delta^{+}\,\mathrm{Sc})\,, \tag{2}$$

which is relation familiar for the mass transfer coefficient according to the film theory, where δ^{+} represents the dimensionless width of the film. Experimental verification of relation (*2*) or (*1*) is very difficult with respect to a considerable length of the entrance section. Experimentally it is usually possible to obtain the mass transfer coefficients averaged over a certain length of the operating section. Thus for verification of the model[1] it is necessary to derive the corresponding relation for the mass transfer coefficients in the entrance section where the concentration profile is not fully developed and the steady state is not reached.

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If mass transfer in the entrance section of length L is studied two situations can be met.

a) Thickness of the concentration boundary layer is smaller than the thickness of the laminar layer ($\delta_c < \delta$) over the whole length L. This means that mass transfer takes place mostly through molecular mechanism and the whole situation should be described as mass transfer into laminar flow of semiinfinite layer. This situation has been studied by Krammers and Kreyger[2] who, for the dimensionless mass transfer coefficient averaged over the length L, have derived the relation

$$k^{+} = 0{\cdot}808\,(\mathrm{Sc}^2 L^{+})^{-1/3}\,. \tag{3}$$

By comparison of results calculated according to relation (*3*) with the experimental data[3] for Sc = 1 160 000 and L^{+} = 8 120 a very good agreement in the range of 2% has been reached.

b) Much more frequent and complex is the situation when in the part or nearly in the whole studied section with the length L is $\delta_c \geqq \delta$. Then mass transfer takes place both due to molecular and turbulent mechanism to a different degree. Let us consider the following mechanism. Liquid with developed velocity profile which is approximately linear in close vicinity of the wall flows along the solid surface from which, from the point $x = 0$ mass is transferred into the liquid stream, see Fig. 1. On the interface is kept the component concentration c^{*} given by solubility of the component and in the turbulent core the concentration c_b. According to the basic assumption of the tested model[1] there exists a laminar layer where mass transfer due only to molecular mechanism takes place in close vicinity of the interface. Across the transition region which is in contact with the laminar layer on one side and with

turbulent core on the other side mass transfer takes place by unsteady diffusion for the time given by the time scale of turbulence in the transition layer τ. For the mean value of the mass transfer coefficient in the transition layer k^* averaged over the time scale of turbulence τ, according to the theoretical model[1] the relation is obtained

$$k^* = 2u^*/(\sqrt{\pi}\,\lambda^+\,\mathrm{Sc}^{1/2})\,, \tag{4}$$

valid for $\mathrm{Sc} > 10$. On basis of the known k^* it is possible, by comparison of mass transfer rates, to obtain the relation

$$k^*(c_x - c_b) = -D\left(\frac{\partial c}{\partial y}\right)_{x,\delta}\,, \tag{5}$$

for the mean concentration on the interface between the laminar and transition layer c_x. The known concentration c_x enables simplification of the mathematical description of the system in Fig. 1 to the description of the situation in the laminar layer in close vicinity of interface.

The mass transfer rate in the system demonstrated in Fig. 1 can be characterized by the dimensionless mass transfer coefficient, which is either local

$$k_x^+ = \frac{-D\left(\frac{\partial c}{\partial y}\right)_{x,0}}{(c^* - c_b)\,u^*}\,, \tag{6}$$

or averaged over the length L

$$k^+ = \frac{1}{L}\int_0^L k_x^+\,\mathrm{d}x\,. \tag{7}$$

When in Eq. (6) x increases beyond all limits, k_x^+ becomes the dimensionless mass transfer coefficient in steady state k_∞^+. Similarly for $L \to \infty$ also k^+ according to Eq. (7) becomes equal to k_∞^+. This means that with increasing x or L the mass transfer coefficients calculated according to relations (1), (6) and (7) are approaching one another.

As it can be seen from Eqs (6) and (7) for evaluation of mass transfer coefficients it is necessary to know the derivation of the concentration profile on the interface in dependence on coordinate x. This derivation can be obtained by solution of the diffusion equation describing the mass transfer in the laminar layer.

The balance of the transported component for the volume of liquid in the laminar layer originated by penetration of layers of thicknesses Δx and Δy with the width s, the differential equation is obtained

$$D\frac{\partial^2 c}{\partial y^2} - ay\frac{\partial c}{\partial x} = 0\,. \tag{8}$$

In its derivation it has been assumed that the effect of axial diffusion is negligible which was later verified by comparison of solutions of Eq. (8) and of the equation including another term $D(\partial^2 c/\partial x^2)$. As in both cases the principle of solution is analogical, only Eq. (8) is being solved here.

The boundary conditions according to Fig. 1 are written in the form

$$x = 0 \quad y \geqq 0 \quad c = c_b\,, \tag{9a}$$

$$x > 0 \quad y = 0 \quad c = c^*\,, \tag{9b}$$

$$x > 0 \quad y = \delta \quad c = c_x\,. \tag{9c}$$

By introduction of new variables it is possible to arrange Eq. (8) into the form suitable for solution and to solve it by the method of separation of variables. For the concentration profile in the laminar layer the relation is then obtained

$$\frac{c - c_b}{c^* - c_b} = 1 - \frac{y^+/\delta^+}{1 + \dfrac{\lambda^+\sqrt{\pi}}{2\delta^+}\,\mathrm{Sc}^{-1/2}} + \\ + 2{\cdot}329375\sum_{i=1}^{\infty} A_i \varrho_i^{1/3}(y^+/\delta^+)^{1/2}\, J_{1/3}\left[\tfrac{2}{3}\varrho_i(y^+/\delta^+)^{3/2}\right] \exp\left(-\varrho_i^2\frac{x^+}{\delta^{+3}\,\mathrm{Sc}}\right). \tag{10}$$

By substitution into relations (6) and (7) for $(\partial c/\partial y)_{x,0}$ from Eq. (10) the relations are obtained for calculation of local dimensionless mass transfer coefficient in the form

$$k_x^+ = \left[(\lambda^+\sqrt{\pi}/2 + \delta^+\,\mathrm{Sc}^{1/2})\,\mathrm{Sc}^{1/2}\right]^{-1} + \frac{1{\cdot}808662}{\delta^+\,\mathrm{Sc}}\sum_{i=1}^{\infty} A_i\varrho_i^{2/3}\exp\left(-\varrho_i^2\frac{x^+}{\delta^{+3}\,\mathrm{Sc}}\right) \tag{11}$$

and of the dimensionless mass transfer coefficient averaged over the dimensionless length L^+

$$k^+ = \left[(\lambda^+\sqrt{\pi}/2 + \delta^+\,\mathrm{Sc}^{1/2})\,\mathrm{Sc}^{1/2}\right]^{-1} - \frac{1{\cdot}808662\delta^{+2}}{L^+}\sum_{i=1}^{\infty} A_i\varrho_i^{-4/3}\,. \\ .\left[\exp\left(-\varrho_i^2\frac{L^+}{\delta^{+3}\,\mathrm{Sc}}\right) - 1\right]. \tag{12}$$

At arrangement of Eqs (*11*) and (*12*) in the relation for derivation $(\partial c/\partial y)_{x,0}$ for $J_{1/3}(t)$ it has been substituted from equation

$$J_{1/3}(t) = 0{\cdot}888823t^{1/3} \qquad (13)$$

which is an approximate relation[4] for the Bessel function for $t \to 0$. This substitution is justified as the argument equal to zero is directly substituted. Relations (*11*) and (*12*) can be moreover arranged to another form by substitution for x^+ ($x^+ = x \,.\, \mathrm{Re} \,.\, \sqrt{(f/2)}/d$ and for $L^+(L^+ = L \,.\, \mathrm{Re} \,.\, \sqrt{(f/2)}/d)$. In the discussed relations appear as new parameters the Reynolds number and the ratio L/d or x/d.

The values of ϱ_i are roots of equation

$$J_{1/3}\left(\tfrac{2}{3}\varrho_i\right) - \frac{\varrho_i}{1 + \dfrac{2\delta^+}{\sqrt{\pi}\,\lambda^+}\,\mathrm{Sc}^{1/2}}\, J_{4/3}\left(\tfrac{2}{3}\varrho_i\right) = 0\,, \quad i = 1, 2, 3, \ldots, \infty \qquad (14$$

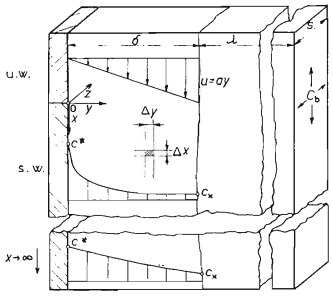

FIG. 1

Dissolution of Solid Component in Liquid Layer in which the Linear Velocity Profile is Fully Developed

u.w. unsoluble wall, s. w. soluble wall.

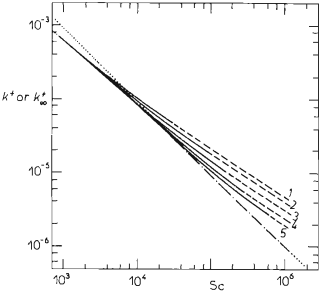

FIG. 2

Dimensionless Mass Transfer Coefficient in Dependence on Schmidt Number

–·–·–·– Kolář[1]; ····· film theory; – – – – – Krammers and Kreyger[2]; ——— relation (*12*) *1* Re = 5000; *2* Re = 10000; *3* Re = = 20000; *4* Re = 50000; *5* Re = 100000; $L/d = 15$.

and the constants A_i can be evaluated according to relation

$$A_i = \left\{ \varrho_i^2 \left[J_{1/3}^2(\tfrac{2}{3}\varrho_i) - \frac{1}{\varrho_i} J_{1/3}(\tfrac{2}{3}\varrho_i) J_{4/3}(\tfrac{2}{3}\varrho_i) + J_{4/3}^2(\tfrac{2}{3}\varrho_i) \right] \right\}^{-1} . \quad i = 1, 2, 3, \ldots, \infty \tag{15}$$

With respect to the fact that the calculations cannot be performed with an infinite number of constants A_i and ϱ_i it has been necessary to neglect in summations the

TABLE I

Values of First Ten Constants ϱ_i and A_i and Sums SUM for Some Values of Schmidt Number

i	Sc = 488 SUM = $9{\cdot}90272 \cdot 10^{-4}$		Sc = 930 SUM = $9{\cdot}90375 \cdot 10^{-4}$		Sc = 4 620 SUM = $9{\cdot}92121 \cdot 10^{-4}$	
	ϱ_i	A_i	ϱ_i	A_i	ϱ_i	A_i
1	2·56947	0·335784	2·74983	0·306935	3·25301	0·259935
2	6·91087	0·146625	7·00793	0·142875	7·37440	0·131196
3	11·5297	0·0897301	11·5909	0·0888105	11·8420	0·0853451
4	16·2014	0·0642372	16·2456	0·0638931	16·4322	0·0625054
5	20·8912	0·0499394	20·9257	0·0497764	21·0731	0·0490988
6	25·5893	0·0408214	25·6175	0·0407321	25·7390	0·0403545
7	30·2917	0·0345089	30·3156	0·0344547	30·4188	0·0342240
8	34·9969	0·0298827	35·0176	0·0298475	35·1072	0·0296965
9	39·7037	0·0263480	39·7220	0·0263238	39·8012	0·0262198
10	44·4117	0·0235598	44·4281	0·0235424	44·4990	0·0234678

i	Sc = 51 900 SUM = $9{\cdot}96731 \cdot 10^{-4}$		Sc = 58 200 SUM = $9{\cdot}96637 \cdot 10^{-4}$		Sc = 88 700 SUM = $9{\cdot}95523 \cdot 10^{-4}$	
	ϱ_i	A_i	ϱ_i	A_i	ϱ_i	A_i
1	3·90359	0·240264	3·92545	0·240062	3·99847	0·239522
2	8·18549	0·117834	8·22199	0·117558	8·34863	0·116767
3	12·5794	0·0784979	12·6207	0·0782629	12·7703	0·0775312
4	17·0636	0·0588681	17·1041	0·0586989	17·2562	0·0581327
5	21·6119	0·0470196	21·6494	0·0469036	21·7943	0·0464934
6	26·2037	0·0390811	26·2377	0·0390015	26·3719	0·0387087
7	30·8249	0·0333976	30·8557	0·0333420	30·9788	0·0331314
8	35·4667	0·0291338	35·4945	0·0290940	35·6072	0·0289400
9	40·1229	0·0258211	40·1483	0·0257919	40·2516	0·0256771
10	44·7898	0·0231760	44·8130	0·0231540	44·9081	0·0230668

terms of higher order. In Eqs (*10*) and (*11*) no difficulties have been met. The decrease of terms in sums is fast and ten constants A_i and ϱ_i usually represent sufficient accuracy. The problems result only from Eq. (*12*) where, when from a certain index the value of the exponential term is neglected remain the terms of the type $-A_i\varrho_i^{-4/3}$. By calculation of a larger number of constants A_i and ϱ_i it has been found out that for these terms is the dependence of A_i on ϱ_i in logarithmic coordinates linear and the remaining terms can be summed up[5].

TABLE I
(*Continued*)

Sc = 6 260 SUM = 9·93014 . 10^{-4}		Sc = 9 030 SUM = 9·94388 . 10^{-4}		Sc = 11 400 SUM = 9·93552 . 10^{-4}		Sc = 14 800 SUM = 9·94695 . 10^{-4}	
ϱ_i	A_i	ϱ_i	A_i	ϱ_i	A_i	ϱ_i	A_i
3·34974	0·254818	3·46293	0·249993	3·53211	0·247593	3·60634	0·245431
7·46603	0·128881	7·58432	0·126226	7·66310	0·124658	7·75357	0·123047
11·9110	0·0844882	12·0045	0·0833961	12·0699	0·0826817	12·1483	0·0818787
16·4853	0·0621325	16·5589	0·0616340	16·6116	0·0612913	16·6761	0·0608876
21·1158	0·0489096	21·1755	0·0486506	21·2188	0·0484681	21·2723	0·0482475
25·7745	0·0402469	25·8245	0·0400977	25·8609	0·0399910	25·9063	0·0398603
30·4492	0·0341574	30·4920	0·0340644	30·5233	0·0339973	30·5625	0·0339144
35·1336	0·0296526	35·1710	0·0295910	35·1985	0·0295463	35·2329	0·0294907
39·8246	0·0261894	39·8578	0·0261466	39·8821	0·0261154	39·9127	0·0260765
44·5200	0·0234460	44·5498	0·0234151	44·5717	0·0233925	44·5992	0·0233643

Sc = 136 000 SUM = 9·93596 . 10^{-4}		Sc = 194 000 SUM = 9·90296 . 10^{-4}		Sc = 687 000 SUM = 9·69524 . 10^{-4}		Sc = 1 160 000 SUM = 9·59404 . 10^{-4}	
ϱ_i	A_i	ϱ_i	A_i	ϱ_i	A_i	ϱ_i	A_i
4·06135	0·239203	4·10585	0·239046	4·21822	0·238840	4·24874	0·238818
8·46339	0·116252	8·54758	0·115978	8·76960	0·115593	8·83175	0·115549
12·9145	0·0769929	13·0254	0·0766795	13·3369	0·0761847	13·4282	0·0761232
17·4113	0·0576669	17·5364	0·0573688	17·9138	0·0568347	18·0309	0·0567592
21·9488	0·0461239	22·0789	0·0458668	22·5001	0·0453432	22·6390	0·0452587
26·5202	0·0384259	26·6492	0·0382151	27·0960	0·0377311	27·2525	0·0376420
31·1184	0·0329170	31·2431	0·0327482	31·7014	0·0323164	31·8716	0·0322263
35·7374	0·0287768	35·8564	0·0286427	36·3162	0·0282657	36·4965	0·0281775
40·3728	0·0255517	40·4853	0·0254451	40·9399	0·0251199	41·1271	0·0250355
45·0208	0·0229692	45·1270	0·0228840	45·5717	0·0226053	45·7633	0·0225258

As results from Eqs (*14*) and (*15*) the constants A_i and ϱ_i are functions of the Schmidt number and of thicknesses δ^+ and λ^+. For the values $\delta^+ = 1$ and $\lambda^+ = 20$ which have been obtained for the pipe of circular cross section[1] and some of Schmidt numbers are the first ten constants A_i and ϱ_i and the value of the sum SUM calculated according to relation

$$\mathrm{SUM} = \sum_{i=1}^{\infty} A_i \varrho_i^{-4/3} \qquad (16)$$

given in Table I. This sum becomes especially significant for small values of L^+ and large values of the Schmidt number when, for the above mentioned assumptions to be met, it is necessary to take into account larger number of constants than ten. For illustration let us consider the following example of calculation: Calculate the value of k^+ for Sc = 930 in the pipe of circular cross section for the length of averaging $L^+ = 10$. Solution is performed according to relation

$$k^+ = \frac{1}{(17{\cdot}724538 + \mathrm{Sc}^{1/2})\,\mathrm{Sc}^{1/2}} - \frac{1{\cdot}808662}{L^+} \cdot$$

$$\cdot \left\{ \sum_{i=1}^{10} A_i \varrho_i^{-4/3} \left[\exp\left(-\varrho_i^2 \frac{L^+}{\mathrm{Sc}} \right) - 1 \right] - \mathrm{SUM} \right\} = 3{\cdot}935 \,.\, 10^{-3}, \qquad (17)$$

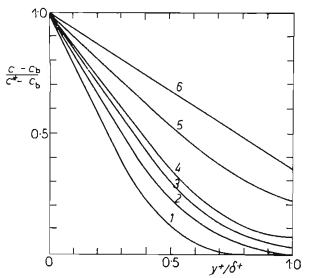

FIG. 3

Concentration Profiles in Laminar Layer in Dependence on Dimensionless Coordinate x^+ (Sc = 930)

1 $x^+ = 15{\cdot}6$; *2* $x^+ = 31{\cdot}2$; *3* $x^+ = 46{\cdot}8$; *4* $x^+ = 78$; *5* $x^+ = 156$; *6* $x^+ = 1561$.

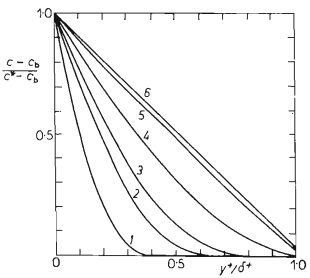

FIG. 4

Concentration Profiles in Laminar Layer in Dependence on Dimensionless Coordinate x^+ (Sc = 88700)

1 $x^+ = 156$; *2* $x^+ = 781$; *3* $x^+ = 1561$; *4* $x^+ = 4683$; *5* $x^+ = 15612$; *6* $x^+ = 46837$.

where the values of constants A_i and ϱ_i and the values of SUM are taken for Sc = 930 from Table I.

DISCUSSION

Equations have been solved numerically on the computer which enabled to obtain a number of useful dependences. The most frequently determined are the dependences of k_∞^+ and k^+ on Sc. These dependences are plotted in Fig. 2. It can be seen from this figure that Eq. (*12*) (solid line) becomes, for large values of Schmidt number and constant ratio L/d equal to Eq. (*3*) (dashed line) derived for the semiinfinite layer. Moreover it is possible to see from Fig. 2 that with changing thickness of the laminar layer, due to the change in the Re number, changes also the value of the Schmidt number, from which it is possible to consider the thickness of the laminar layer as semiinfinite space in comparison with the thickness of the concentration boundary layer. For example for Re = 5000 is the value Sc ≈ 20 000, for Re = = 10000, Sc ≈ 100000 *etc.* With decreasing value of the Schmidt number decreases also the dependence of k^+ on Re number, so that finally in the region Sc < 10^3 the independence of k^+ on Re is obtained. The relation (*12*) denoted by the solid line takes the form of Eq. (*1*) denoted by the dashed and dotted curves, from which

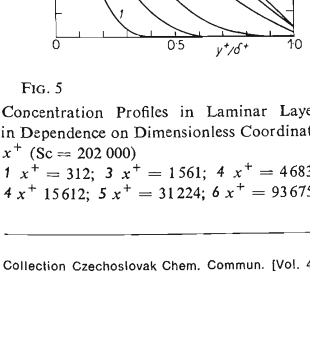

FIG. 5

Concentration Profiles in Laminar Layer in Dependence on Dimensionless Coordinate x^+ (Sc = 202 000)
1 x^+ = 312; *3* x^+ = 1561; *4* x^+ = 4683; *4* x^+ 15612; *5* x^+ = 31224; *6* x^+ = 93675.

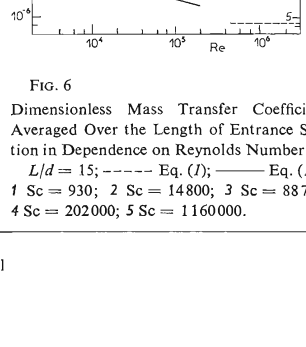

FIG. 6

Dimensionless Mass Transfer Coefficient Averaged Over the Length of Entrance Section in Dependence on Reynolds Number
L/d = 15; ----- Eq. (*1*); ——— Eq. (*12*); *1* Sc = 930; *2* Sc = 14800; *3* Sc = 88700; *4* Sc = 202000; *5* Sc = 1160000.

is this mentioned dependence obvious. In Fig. 2 is also plotted the dependence of the mass transfer coefficient on Schmidt number according to the film theory as the dotted line. From this figure is obvious that the difference between the theory by Kolář[1] (Eq. (*1*)) and the film theory (Eq. (*2*)) is for $Sc = 10^6$ already negligible. The formation of the concentration boundary layer can be observed in Figs 3 to 5 where the profiles of concentrations in the laminar layer are plotted in dependence on the dimensionless coordinate x^+. From Fig. 3 is obvious that for the value of Schmidt number $Sc = 930$ the diffusion coefficient is sufficiently large so that the steady concentration profile develops nearly immediately in comparison with Figs 4 and 5 for $Sc = 88700$ and $Sc = 202000$ where for reaching of the steady state is necessary by orders of magnitude longer distance, which of course affects the mass transfer coefficient averaged over the length of this entrance section.

The entrance section lengths for which the mass transfer coefficients k_x^+ and k^+ reach their steady values can be calculated on basis of the formulated model. In the calculation has been assumed that the entrance section length is such for which the change in k_x^+ and k^+ caused by its continuing prolongation is smaller than 1%, 3% or 5%. The corresponding entrance region lengths x^+ for k_x^+ and L^+ for k^+ are then given for various values of the Schmidt number in Table II. It can be seen from this Table that the entrance section lengths are significantly increasing with the increasing value of the Schmidt number. So *e.g.* for the maximum change in the mass transfer coefficients by 1% and for $Sc = 6260$, the steady value of k_x^+ is reached for $x^+ = 2699$ which for $Re = 10000$ represents the ratio $x/d = 4{\cdot}3$ and k^+ for $L^+ = 70459$, which represents the ratio $L/d = 112{\cdot}1$. For $Sc = 687000$ is $x^+ = 183264$ which for $Re = 10000$ represents the ratio $x/d = 291{\cdot}4$ and $L^+ = 4447404$

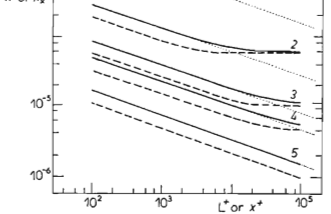

FIG. 7

Dimensionless Mass Transfer Coefficients k_x^+ and k^+ in Dependence on Dimensionless Coordinate x^+ and L^+

····· Krammers and Kreyger[2]; – – – – Eq. (*11*) $k_x^+ = f(x^+)$; ——— Eq. (*12*) $k^+ = f(L^+)$; *1* $Sc = 930$; *2* $Sc = 14800$; *3* $Sc = 88700$; *4* $Sc = 202000$; *5* $Sc = 1160000$.

and thus $L/d = 7072$. From this example results that experimental measurements of k^+ would perhaps not be possible for Sc = 687000 with regard to the magnitude of ratios x/d and L/d.

Better than from Fig. 2 is the dependence of k^+ on Re obvious from Fig. 6. It is possible to see from this figure that with increasing value of the Schmidt number it is necessary to increase significantly the value of Re number so that the effect of the entrance section would vanish and k^+ given by Eq. (*12*), which is in the figure denoted by solid line, would become identical with k_∞^+ given by Eq. (*1*) which is in the same figure denoted by dashed line. The necessary increase in Re number is due to the fact that with increasing value of Sc decreases the thickness of the concentration boundary layer and this decrease must be counterbalanced by decrease in the thickness of the laminar layer and thus by increase in Re.

The effect of the length of the entrance section on values k_x^+ and k^+ according to Eqs (*11*) and (*12*) is discussed in Fig. 7, where k_x^+ as $f(x^+)$ is denoted by dashed line and k^+ as $f(L^+)$ is denoted by solid line. In this figure the effect of Re number is included in the dimensionless variables x^+ and L^+. From this figure is obvious that with increasing value of x^+ or L^+ the values k_x^+ and k^+ are approaching each

TABLE II

Dimensionless Lengths for which the Mass Transfer Coefficient Reaches the Steady Value in Dependence on Schmidt Number

Sc	1%		3%		5%	
	x^+	L^+	x^+	L^+	x^+	L^+
480	387	14 985	308	4 995	270	2 997
930	633	21 190	498	7 063	435	4 238
4 620	2 126	56 814	1 646	18 938	1 423	11 363
6 260	2 699	70 459	2 087	23 486	1 802	14 092
9 030	3 622	92 468	2 795	30 823	2 410	18 494
11 400	4 383	110 660	3 379	36 887	2 912	22 132
14 800	5 444	136 102	4 194	45 367	3 613	27 220
51 900	16 194	395 490	12 452	131 830	10 712	79 098
58 200	17 954	438 070	13 804	146 023	11 875	87 614
88 700	26 355	641 520	20 260	213 840	17 426	128 304
136 000	39 153	951 676	30 095	317 225	25 883	190 335
194 000	54 636	1 327 110	41 994	442 370	36 115	265 422
687 000	183 264	4 447 404	140 847	1 482 467	121 124	889 481
1 160 000	305 004	7 401 066	234 408	2 467 021	201 582	1 480 212

other and as results from relations (6) and (7) for x^+ and $L^+ \to \infty$ both values are becoming identical and equal to k_∞^+. The character of the dependence is again affected by the value of Sc number. It is also worth stating that for small values of x^+ and L^+ the thickness of the concentration boundary layer is sufficiently small so that the thickness of the laminar layer in comparison with it could be considered to be a semiinfinite space and the mass transfer coefficients could be expressed on basis of models by Krammers and Kreyger[2]. These comparisons are also made in Fig. 7, where the values of the mass transfer coefficient averaged over the length of the entrance section calculated on basis of the models by Krammers and Kreyger[2] are denoted by dotted line.

LIST OF SYMBOLS

a	proportionality constant in Eq. (8) (s^{-1})
A_i	constant defined by Eq. (15)
c	concentration (mol m^{-3})
c^*	concentration of component on interface (mol m^{-3})
c_b	concentration of component in the bulk of liquid phase (mol m^{-3})
c_x	concentration defined by Eq. (5) (mol m^{-3})
d	diameter of pipe (m)
D	diffusivity of component ($m^2 s^{-1}$)
$f = 0{\cdot}078\ \mathrm{Re}^{-0.25}$	friction factor
$J_\nu(x)$	Bessels function of ν order
k	mass transfer coefficient averaged over length L ($m s^{-1}$)
$k^+ = k/u^*$	
k_∞	fully developed mass transfer coefficient ($m s^{-1}$)
$k_\infty^+ = k_\infty/u^*$	
k_x	local mass transfer coefficient ($m s^{-1}$)
$k_x^+ = k_x/u^*$	
k^*	mass transfer coefficient in transition region ($m s^{-1}$)
L	length of entrance section (m)
$L^+ = L u^*/\nu$	
$\mathrm{Re} = du/\nu$	Reynolds number
s	width (m)
SUM	sum defined by Eq. (16)
$\mathrm{Sc} = \nu/D$	Schmidt number
u	component of average velocity in direction of flow ($m s^{-1}$)
$u^* = u\sqrt{f/2}$	friction velocity ($m s^{-1}$)
x	coordinate (m)
$x^+ = x u^*/\nu$	
y	coordinate (m)
$y^+ = y u^*/\nu$	
z	coordinate (m)
δ	thickness of laminar layer (m)
$\delta^+ = \delta u^*/\nu$	
δ_c	thickness of concentration boundary layer (m)

λ	thickness of transition layer (m)
$\lambda^+ = \lambda u^*/\nu$	
ν	kinematic viscosity ($m^2 s^{-1}$)
ϱ_i	constant defined by Eq. (*14*)
$\tau = \lambda^2/\nu$	time scale of turbulence in transition region (s)

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